

POWER TO DETECT DIFFERENCES AND TRENDS IN APPARENT SURVIVAL RATES¹

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Abstract. A major goal of demographic monitoring is the detection of trends in vital rates or differences in vital rates between populations. Here we apply Cormack-Jolly-Seber models to simulated capture-recapture data to assess the statistical power to detect trends or differences in adult apparent survival rates. Simulations were based on the complete range of parameter values, from 0.1 to 0.9 for survival and from 0.05 to 0.95 for recapture probability. Sample sizes needed to detect the smallest effect size (5%) with 80% power were large, ranging from hundreds to thousands of individuals released per year, depending on the alpha-level and combination of survival and recapture probabilities. Larger effect sizes were relatively easily detected, with effect sizes of 15% and higher requiring from just a few individuals to tens of individuals released per year. Power increased with higher survival and recapture rates, with the largest gains achieved by increasing p at the low end of its range, particularly for $p = 0.05$ to 0.35.

Key words: Cormack-Jolly-Seber models, demographic monitoring, landbirds, power analysis, survival rate estimation.

PODER PARA DETECTAR DIFERENCIAS Y TENDENCIAS EN TASAS DE SOBREVIVENCIA APARENTE

Resumen. Una meta principal del monitoreo de poblaciones es la detección de tendencias en tasas vitales o diferencias en tasas vitales entre poblaciones. Aquí aplicamos modelos de Cormack-Jolly-Seber para simular datos de marcaje-recaptura y estimar el poder estadístico para detectar tendencias o diferencias en tasas de sobrevivencia aparente. Las simulaciones se basaron en el rango completo de valores paramétricos, entre 0.1 y 0.9 para sobrevivencia y entre 0.05 y 0.95 para la probabilidad de recaptura. Los tamaños de muestra necesarios para detectar el efecto mínimo (5%) con poder del 80% fueron grandes, entre cientos y miles de individuos marcados por año, dependiendo del nivel de alfa y la combinación de probabilidades de sobrevivencia y recaptura. Efectos mayores fueron detectados con relativa facilidad, con efectos de 15% y más pudiendo ser detectados con unos pocos individuos a algunas decenas. El poder aumentó con tasas de sobrevivencia y recaptura más altas, con las mejoras más importantes obtenidas mediante el aumento de p en el extremo inferior de su rango, particularmente para $p = 0.05$ a 0.35.

Palabras clave: modelos Cormack-Jolly-Seber, monitoreo demográfico, aves terrestres, análisis de poder, estimación de tasas de sobrevivencia.

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INTRODUCTION

Although a focus on abundance and trends of animal populations is of fundamental interest to models of population dynamics, it is also essential to acquire information about the biological processes of survival, reproduction, and movement that are responsible for changes in abundance (Williams et al. 2002). The increasing emphasis in population dynamics models on the estimation of primary demographic parameters, and on quantifying variability in these parameters, has led to the development of various demographic monitoring efforts for birds, including the British Constant Effort Sites (CES) scheme (Peach et al. 1996) and the North American Monitoring Avian Productivity and Survivorship (MAPS) program (DeSante et al. 1995), as well as for other animals (e.g., Burnham et al. 1987). A major goal of demographic monitoring is the detection of differences in vital rates between populations or trends in vital rates over time. Achievement of this goal necessitates consideration of statistical power, or the ability to reject null hypotheses when they are false.

Among the principal vital rates addressed by demographic monitoring is the annual apparent survival rate of adult animals (hereafter "survival"). Because survival is a population parameter that often incorporates a large amount of stochasticity (accidental deaths), its underlying sample distribution tends to have high variance; indeed, the capture-mark-recapture (CMR) models used in estimating survival are often thought of as "data-hungry." It is appropriate, therefore, to inquire as to how many animals must be captured, marked, and released back into the population each sampling period in order to have sufficient power to detect differences or trends in survival. Moreover, many authors have stressed the importance of making such inquiries before implementing a demographic monitoring study, particularly one involving estimations of survival (e.g., Cohen 1988, Thomas 1997).

Pollock et al. (1990) presented comparisons of the precision of survival estimates that would be obtained for a range of population sizes, survival and recapture probabilities, and sampling events, but these cannot easily be

converted to power estimates. More recently, the power of between-model likelihood ratio tests can be computed from the computer program POWER (www.mbr-pwrc.usgs.gov/software.html). This program, however, requires the user to generate data by computing expected values under the alternative model for specified sample sizes, to compute estimates under the alternative and null models, and to compute a likelihood-ratio test between models that will provide the χ^2 value and degrees of freedom needed for the power analysis.

Here we apply Cormack-Jolly-Seber models to simulated capture-mark-recapture data to assess the statistical power to detect differences in survival between populations or linear trends in survival, given 20 sampling periods (i.e., years of data for monitoring annual survival) and a range of survival rates, recapture probabilities, effect sizes, and numbers of animals released. We present results in graphical form so that the numbers of animals that must be released each sampling period for each combination of survival and recapture probabilities and effect size can easily be determined.

METHODS

We investigated the statistical power to detect (1) differences in survival between populations and (2) linear declines in survival over time for a range of survival and recapture probabilities. We considered the relatively complete range of survival probabilities (from 0.1 to 0.9 at intervals of 0.1) and recapture probabilities (from 0.05 to 0.95 at intervals of 0.15). We report results for two alpha-levels (0.10 and 0.20) reported by other studies that have evaluated power of monitoring data (e.g., Bart et al. 2004). We considered five effect sizes between 5 and 25%. For the two-population scenario (1 above), effect sizes represent differences in survival between two populations that begin with equal population sizes. For the linear decline scenario (2 above), effect sizes represent incremental proportional changes in survival that would halve the population in the same number of years as a population whose survival initially declined by that effect size and then remained constant. For example, a population with an initial survival rate of 0.50 and a decrease in survival of 5% after the initial time period (to

0.475) would halve in 28 years (i.e., year 28 was the first year for which the population size was equal to or less than 50% of its initial value). The corresponding linear "5%" decline (that would exactly halve the population in 28 years) would be a proportional annual decrease in survival of 0.00373. Our calculations assume initially stable populations (i.e., $\lambda = 1$) and constant recruitment at a level that balances losses at the start of the study. The number of years needed to halve populations under each scenario (given the assumed constant recruitment rate) and annual proportional changes used for the linear decline models are given in Table 1.

For each scenario (two populations or a single declining population) and combination of survival and recapture probabilities, we simulated capture-mark-recapture data sets of various sizes using the deterministic mode of program GENCAPH1 (www.mbr-pwrc.usgs.gov/software.html) for 20 capture periods (i.e., years for estimates of annual survival). For all sets of simulations we chose an initial sample size of 10 individual marked animals released per year. We repeated this process for a series of sample sizes ranging from 20-2000 annual releases of marked animals. We input simulated capture histories into Program MARK (White and Burnham 1999) and estimated survival with Cormack-Jolly-Seber (CJS) models representing null hypotheses (equal survival between populations or time-constant survival) and with CJS models representing "true" alternative hypotheses (Pollock et al. 1990). Power was calculated for each scenario, combination of initial survival rate and recapture probability, and sample size, by entering χ^2 values (and their associated degrees of freedom) from a likelihood-ratio test comparing null and true models into program POWER (www.mbr-pwrc.usgs.gov/software.html).

We plotted power curves (i.e., sample size vs. power) to determine numbers of individuals needed to be released annually to achieve 80% power of rejecting null hypotheses (i.e., $1 - \beta = 0.80$). Needed sample sizes were calculated using inverse prediction (i.e., we predicted the x values at $y = 0.80$) from a line connecting the two power estimates that bracketed power of 0.80. Because power curves were concave (particularly near 0.80 power), predicted needed sample sizes from straight lines are biased slightly high.

RESULTS

The numbers of individual animals needed to be released at each capture period to detect differences in survival between populations (Fig. 1) or trends in survival (Fig. 2) with 80% power are presented as a function of effect size for each combination of survival rate, recapture probability, and α -level (see Appendix). Because needed numbers range variously from <10 to >2000, we present them on a log scale (Fig. 1 and 2). The slight increase in numbers needed to detect larger linear declines (25 and 20%) for species with high survival rates (0.9 and 0.8; Fig. 2 and italicized numbers in the Appendix) was an artifact of calculating linear effect sizes from exact 50% population declines and forcing them to occur over year integers when the number of years to halve the population was small (e.g., 3 or 4 years) and the percent of the population remaining after halving was substantially <50% (Table 1).

Sample sizes needed to detect the smallest effect sizes (5%) with 80% power were large, ranging from hundreds to thousands of individuals released per year, depending on the alpha-level and combination of survival rates and recapture probabilities. Larger effect sizes were relatively easily detected, with effect sizes of 15% and higher requiring from just a few individuals to tens of individuals released per year. The ability to reject null hypotheses was strongly dependent on recapture probabilities; the biggest gains in power came from increasing p at the low end of the range, particularly $p = 0.05$ -0.35. The ability to reject null hypotheses was also dependent on survival rates themselves; again, the largest increase in power arose from increased survival rates at the low end of the range, but the proportional gains in power from higher survival rates were not as great as from increased recapture probabilities. Sample sizes required to detect large effect sizes were typically smaller for the linear decline models than for the two-population comparisons, while small effect sizes were more easily detected in the two-population scenarios. Overall, from all of the values presented in the Appendix and shown in Figs. 1 and 2, the sample sizes needed to detect effect sizes with 80% power at $\alpha = 0.1$ averaged 37% greater than the sample sizes needed at $\alpha = 0.2$.

TABLE 1. Summary of effect sizes for each survival rate considered in simulations.

Higher ϕ (or starting ϕ for linear decline scenarios) ^a	Effect size -- % difference in ϕ (or "% change in ϕ ") ^b	Lower ϕ ^c	No. years (t) to halve population with lower ϕ	% of population remaining after t years	Prop. annual change for linear decline scenarios
0.9	5	0.855	16	48	0.00644
	10	0.810	8	47	0.02771
	15	0.765	5	48	0.07799
	20	0.720	4	45	0.13054
	25	0.675	3	47	0.26328
0.8	5	0.760	17	50	0.00642
	10	0.720	9	47	0.02442
	15	0.680	6	46	0.05911
	20	0.640	4	50	0.15007
	25	0.600	4	41	0.30614
0.7	5	0.665	20	49	0.00527
	10	0.630	10	48	0.02249
	15	0.595	7	46	0.04874
	20	0.560	5	47	0.10411
	25	0.525	4	46	0.17654
0.6	5	0.570	23	50	0.00463
	10	0.540	12	48	0.01801
	15	0.510	8	47	0.04315
	20	0.480	6	46	0.08204
	25	0.450	5	44	0.12511
0.5	5	0.475	28	49	0.00373
	10	0.450	14	49	0.01579
	15	0.425	9	50	0.04087
	20	0.400	7	48	0.07153
	25	0.375	6	45	0.10186
0.4	5	0.380	35	49	0.00297
	10	0.360	17	50	0.01332
	15	0.340	12	48	0.02803
	20	0.320	9	47	0.05274
	25	0.300	7	48	0.09344
0.3	5	0.285	46	50	0.00229
	10	0.270	23	50	0.00961
	15	0.255	16	48	0.02076
	20	0.240	12	48	0.03887
	25	0.225	9	50	0.07444
0.2	5	0.190	69	50	0.00152
	10	0.180	35	49	0.00616
	15	0.170	23	50	0.01498
	20	0.160	17	50	0.02889
	25	0.150	14	49	0.04462
0.1	5	0.005	120	50	0.00074
	10	0.090	69	50	0.00315
	15	0.085	46	50	0.00740
	20	0.080	35	49	0.01333
	25	0.075	28	49	0.02183

^a Higher (two-population scenario) or initial (linear decline scenario) survival rate.

^b Percent difference in survival rate (two-population scenario) or percent change in survival rate (linear decline scenario).

^c Survival rate of the population with lower survival (two-population scenario).

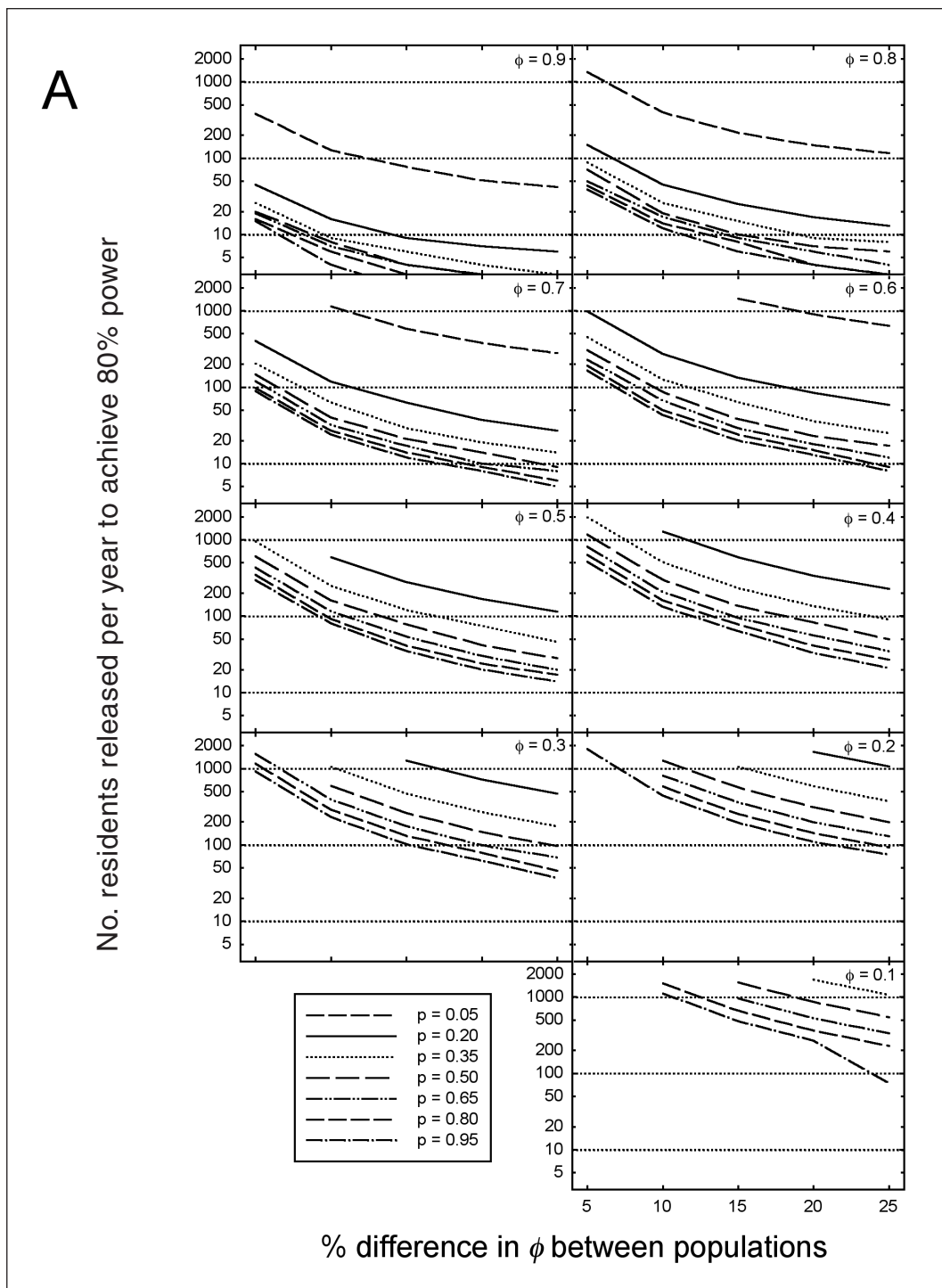


FIGURE 1. Numbers of resident adult birds needed to be released per year to achieve 80% power to detect differences in survival (ϕ) between populations for birds with annual apparent survival rates between 0.10 and 0.90 and α -levels of 0.10 (A) and 0.20 (B).

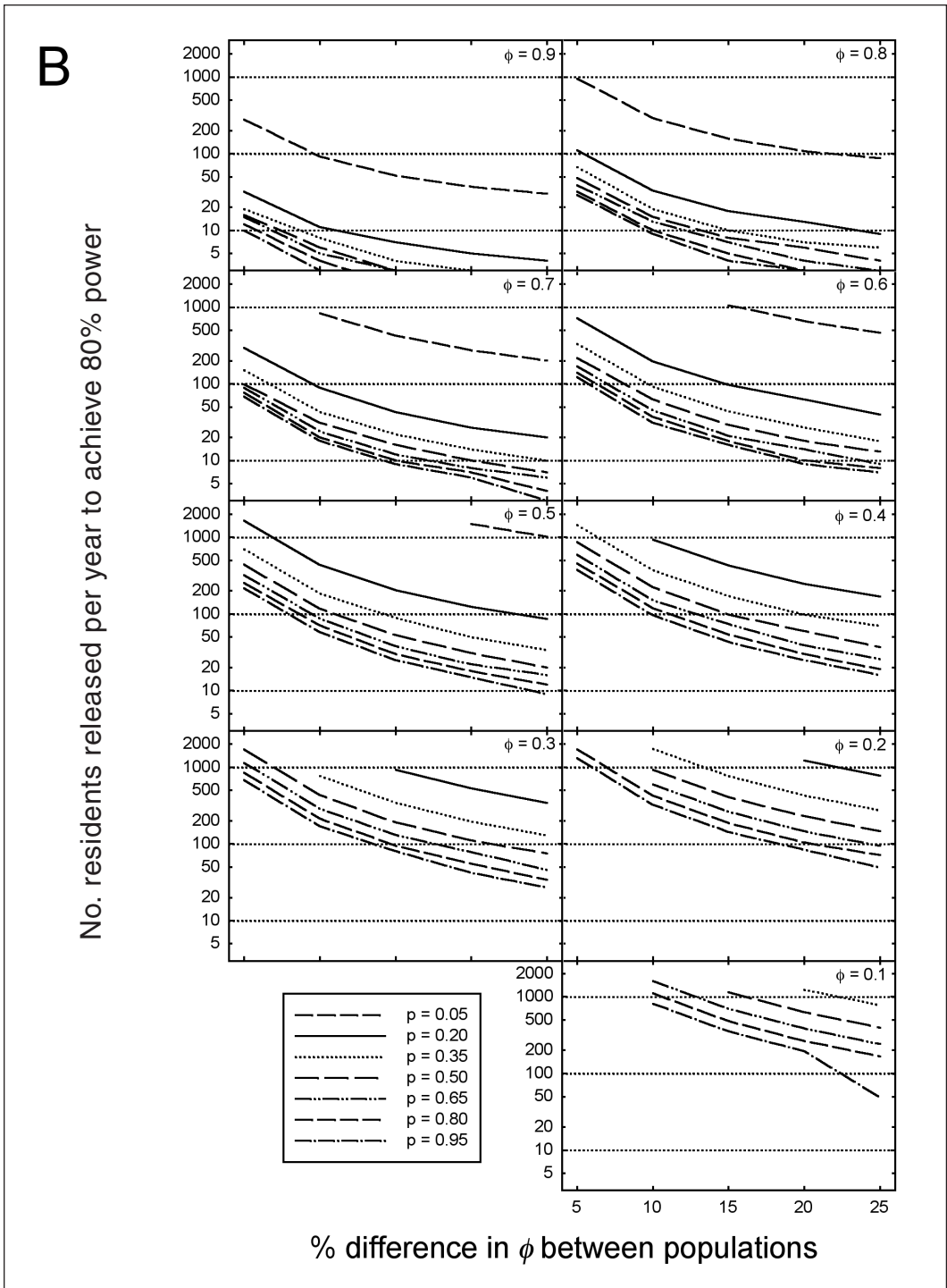


FIGURE 1. Continued.

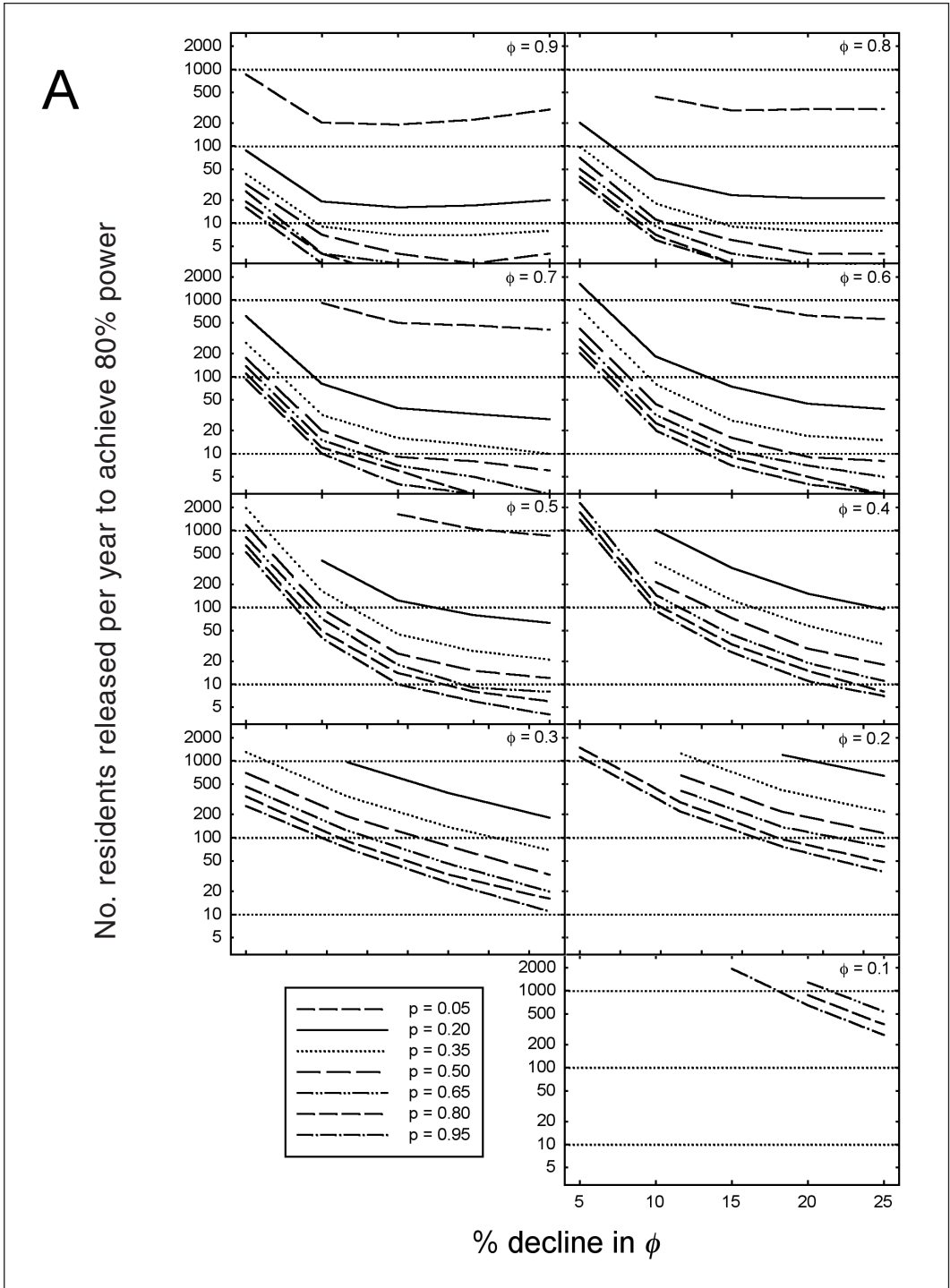


FIGURE 2. Numbers of resident adult birds needed to be released per year to achieve 80% power to detect linear declines in survival (ϕ) for birds with annual apparent survival rates between 0.10 and 0.90 and α -levels of 0.10 (A) and 0.20 (B).

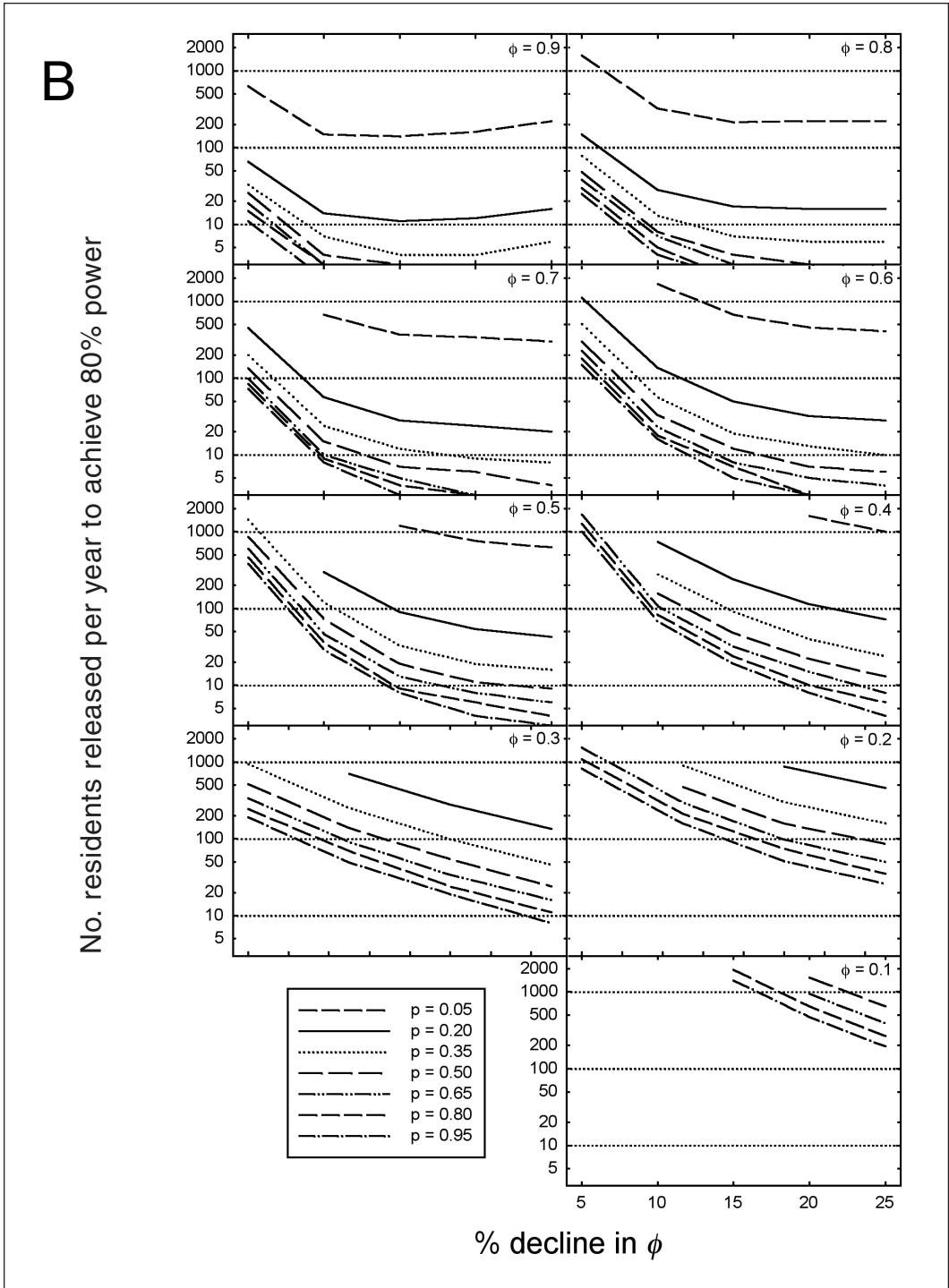


FIGURE 2. Continued.

DISCUSSION

Our power analyses demonstrate that large sample sizes are needed to detect small (5-10%) differences in survival between populations or linear declines in survival in a single population, but that larger effect sizes (15-25%) can be detected with relatively modest sample sizes. Although the larger effect sizes could halve population sizes over very short time intervals, at least for species with higher survival rates, differences of this magnitude are often observed in avian survival data. For example, the largest effect size (25%) is similar to the mean maximum difference in survival rates between MAPS regions for 89 species for which 10-yr time-constant estimates of survival were available from multiple regions (mean maximum difference = 23%; DeSante and Kaschube 2006). Indeed, analyses of MAPS data suggest that differences and declines in survival of the magnitude of differences typically seen between MAPS regions or between clusters of MAPS stations can be detected with 80% power for 84 and 105 species, respectively, at the continental scale, and for 40 and 47 species, respectively, in the Northwest region (the region with the largest number of MAPS stations; DeSante and Saracco 2009).

Adult apparent survival rates of landbirds obtained from pooling CMR data from monitoring stations operated for 6-9 d over three-month periods typically range from about 0.4 (or slightly lower for very small-bodied species such as kinglets (*Regulus* spp.) to 0.6 (or up to nearly 0.7 for jay-sized passerines; DeSante and Kaschube 2009). Clearly, differences and trends in survival are easier to detect for larger species with higher survival rates than for smaller species. For example, differences between two populations can be detected with 80% power at $\alpha = 0.2$ for a species with $\phi = 0.6$ and $p = 0.15$ (a low recapture rate) by releasing 97 birds per year. One would need to release about 429 birds per year to obtain the same result for a species with $\phi = 0.4$ and $p = 0.15$; or would need to increase p to 0.50 to get the same result by releasing 97 birds per year for a species with $\phi = 0.4$.

The largest and most efficient gains in power are provided by increases in p , especially at its lower range of values (e.g., 0.15 to 0.35). Gains provided by increasing p at its higher range of

values (e.g., 0.65 to 0.95) tend to be relatively limited in extent. This suggests that intensive color-band resighting efforts, which typically produce p -values of 0.95 or greater, run simultaneously with constant-effort-type monitoring schemes, would not greatly increase power for those species, which are often ground- or shrub-inhabiting species, for which \hat{p} is already often >0.5 .

In nature, the population sizes of animals that must be sampled to provide 80% power to detect differences and trends in survival are likely to be somewhat greater than indicated here for several reasons. First, transient individuals of the species targeted occur on most areas where animals are sampled. These individuals, which in landbirds can be floaters that have not yet acquired a breeding territory or mate, failed breeders searching for new territories or mates, or post-breeding individuals dispersing to molting or pre-migration staging areas, have essentially zero probability of still being present in the next sampling period and thus have an expected $\phi = 0.0$. Thus, they cannot be counted in the number of "resident" animals needed to detect differences or trends in survival. CMR models are available that can correct for the existence of these transients (Pradel et al. 1997, Nott and DeSante 2002, Hines et al. 2003). Additional sources of heterogeneity deriving from spatial and temporal variation in survival and recapture probability (e.g., due to environmental variation, age/sex, or other behavioral effects) could further lower precision of parameter estimates and result in the need for larger samples than those reported here. Nevertheless, given the difficulty of incorporating all such sources of variation in the planning stages of a study, we feel that the guidelines presented here provide a reasonable starting point for designing capture-recapture studies and for evaluating the likely efficacy of existing studies for detecting effect sizes that are meaningful for a particular population.

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APPENDIX. Sample sizes (numbers of marked animals released per year) needed to detect effect sizes from 5-25% for varying apparent survival rates (ϕ) and recapture probabilities (p). Data are presented graphically in Figures 1 and 2. Sample sizes >2000 are indicated with “—”.

	% difference in ϕ between populations																			
	$\alpha = 0.10$						$\alpha = 0.20$													
	5%	10%	15%	20%	25%		5%	10%	15%	20%	25%									
$\phi = 0.90$	% decline in ϕ																			
	$\alpha = 0.10$						$\alpha = 0.20$													
	5%	10%	15%	20%	25%		5%	10%	15%	20%	25%									
p=0.05	381	127	77	51	42	279	92	52	37	30	857	203	191	221	300	627	148	140	161	221
p=0.20	45	16	9	7	6	32	11	7	5	4	88	19	16	17	20	66	14	11	12	16
p=0.35	26	9	6	4	3	19	8	4	3	2	44	9	7	7	8	33	7	4	4	6
p=0.50	20	8	4	3	3	16	6	3	2	2	32	7	4	3	4	26	4	3	3	3
p=0.65	19	7	4	3	2	15	5	3	2	2	26	4	3	2	2	19	3	2	2	2
p=0.80	16	6	3	2	2	12	4	2	2	2	19	4	2	2	2	15	3	2	2	2
p=0.95	15	4	2	2	2	10	3	2	2	2	16	3	2	2	2	11	2	2	2	2
$\phi = 0.80$	% decline in ϕ																			
	$\alpha = 0.10$						$\alpha = 0.20$													
	5%	10%	15%	20%	25% <td></td> <td>5%</td> <td>10%</td> <td>15%</td> <td>20%</td> <td>25%</td>		5%	10%	15%	20%	25%									
p=0.05	1338	396	215	147	116	953	290	157	108	87	—	442	291	303	303	1599	323	213	222	222
p=0.20	150	45	25	17	13	111	33	18	13	9	202	38	23	21	21	149	28	17	16	16
p=0.35	88	26	15	9	8	67	19	10	7	6	98	18	9	8	8	78	13	7	6	6
p=0.50	71	19	10	7	6	48	15	8	6	4	71	11	6	4	4	48	8	4	3	3
p=0.65	50	17	9	6	4	39	13	7	4	3	51	9	4	3	3	38	7	3	2	2
p=0.80	44	14	8	4	3	32	10	5	3	2	40	7	3	2	2	30	5	2	2	2
p=0.95	39	12	6	4	3	29	9	4	3	2	34	6	3	2	2	25	4	2	2	2
$\phi = 0.70$	% decline in ϕ																			
	$\alpha = 0.10$						$\alpha = 0.20$													
	5%	10%	15%	20%	25% <td></td> <td>5%</td> <td>10%</td> <td>15%</td> <td>20%</td> <td>25%</td>		5%	10%	15%	20%	25%									
p=0.05	—	1142	583	378	278	—	828	425	275	201	—	912	503	464	412	—	665	367	338	300
p=0.20	406	119	63	37	27	296	89	43	27	20	620	81	39	33	28	451	57	28	24	20
p=0.35	205	63	29	19	14	151	43	22	14	10	278	32	16	13	10	200	24	12	9	8
p=0.50	147	40	21	14	9	99	31	16	10	7	176	20	9	8	6	134	15	7	6	4
p=0.65	120	32	17	10	8	89	24	12	8	6	137	15	7	5	3	99	10	5	3	3
p=0.80	98	27	14	9	6	77	20	10	7	4	110	12	6	3	2	84	9	4	3	2
p=0.95	89	24	12	8	5	68	18	9	6	3	93	10	4	3	2	72	8	3	2	2

APPENDIX. Continued.

	% difference in ϕ between populations										% decline in ϕ															
	$\alpha = 0.10$					$\alpha = 0.20$					$\alpha = 0.10$					$\alpha = 0.20$										
	5%	10%	15%	20%	25%	5%	10%	15%	20%	25%	5%	10%	15%	20%	25%	5%	10%	15%	20%	25%						
$\phi = 0.60$																										
p=0.05	—	—	1454	900	636	—	—	1058	657	463	—	—	—	912	627	560	—	1676	666	455	406	—	—	—	—	—
p=0.20	989	271	134	85	59	718	197	97	63	40	1623	184	75	45	38	1124	135	50	32	28	—	—	—	—	—	
p=0.35	452	126	64	36	25	332	92	44	27	18	755	80	27	17	15	505	56	19	13	10	—	—	—	—	—	
p=0.50	308	86	38	23	17	218	62	29	18	13	422	44	16	9	8	301	33	12	7	6	—	—	—	—	—	
p=0.65	230	67	29	18	12	169	45	21	14	9	308	32	11	7	5	226	23	8	5	4	—	—	—	—	—	
p=0.80	191	50	24	15	9	140	37	18	10	8	245	25	9	5	3	180	18	7	3	3	—	—	—	—	—	
p=0.95	166	43	20	13	8	122	31	16	9	7	203	20	7	4	3	149	16	5	3	1	—	—	—	—	—	
$\phi = 0.50$																										
p=0.05	—	—	—	—	1389	—	—	—	1492	1014	—	—	1638	1038	857	—	—	1192	755	625	—	—	—	—	—	
p=0.20	—	593	280	168	115	1641	434	205	124	86	—	408	122	79	63	—	298	90	54	43	—	—	—	—	—	
p=0.35	953	250	121	75	46	699	185	89	50	34	1978	162	45	27	21	1441	120	33	19	16	—	—	—	—	—	
p=0.50	610	161	78	42	28	441	117	53	31	20	1182	95	25	15	12	862	73	19	11	9	—	—	—	—	—	
p=0.65	433	116	54	30	20	320	87	38	22	16	823	71	18	9	8	603	47	13	8	6	—	—	—	—	—	
p=0.80	348	92	41	24	17	254	71	30	18	12	642	49	14	8	6	465	36	9	6	4	—	—	—	—	—	
p=0.95	295	81	35	20	14	216	58	25	15	9	525	40	10	6	4	381	29	8	4	3	—	—	—	—	—	
$\phi = 0.40$																										
p=0.05	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1375	—	—	—	1594	999	—	—	—	—	—	
p=0.20	—	1282	589	340	228	—	934	429	247	168	—	1020	327	151	94	—	738	239	113	73	—	—	—	—	—	
p=0.35	1976	509	234	136	91	1442	371	171	98	70	—	384	124	58	33	—	280	91	40	24	—	—	—	—	—	
p=0.50	1179	299	136	83	50	863	223	98	60	37	—	214	73	29	18	—	157	48	22	13	—	—	—	—	—	
p=0.65	816	208	95	56	35	594	151	74	39	26	2292	145	44	19	11	1673	107	32	15	8	—	—	—	—	—	
p=0.80	633	161	78	41	27	458	119	54	30	19	1726	110	33	15	8	1259	83	24	10	6	—	—	—	—	—	
p=0.95	517	133	64	33	21	378	96	43	25	16	1382	89	26	11	7	1008	68	19	8	4	—	—	—	—	—	

APPENDIX. Continued.

	% difference in ϕ between populations										% decline in ϕ														
	$\alpha = 0.10$					$\alpha = 0.20$					$\alpha = 0.10$					$\alpha = 0.20$									
	5%	10%	15%	20%	25%	5%	10%	15%	20%	25%	5%	10%	15%	20%	25%	5%	10%	15%	20%	25%					
$\phi = 0.30$																									
p=0.05	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
p=0.20	—	—	1274	721	469	—	—	—	922	528	343	—	—	—	—	—	—	—	—	—	—	—	—	—	—
p=0.35	—	—	1059	473	270	176	—	—	770	344	196	129	—	—	—	—	—	—	—	—	—	—	—	—	—
p=0.50	—	—	591	263	147	96	1716	431	190	111	75	75	—	—	—	—	—	—	—	—	—	—	—	—	—
p=0.65	1570	392	177	99	69	69	1144	287	131	78	46	46	—	—	—	—	—	—	—	—	—	—	—	—	—
p=0.80	1164	290	132	79	46	46	849	213	95	55	34	34	—	—	—	—	—	—	—	—	—	—	—	—	—
p=0.95	926	232	102	62	37	37	676	170	80	42	27	27	—	—	—	—	—	—	—	—	—	—	—	—	—
$\phi = 0.20$																									
p=0.05	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
p=0.20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
p=0.35	—	—	—	—	—	1668	1069	—	—	—	1215	775	—	—	—	—	—	—	—	—	—	—	—	—	—
p=0.50	—	—	—	—	—	1060	590	376	—	—	1736	767	430	274	—	—	—	—	—	—	—	—	—	—	—
p=0.65	—	—	—	—	—	1277	565	312	199	—	926	406	229	146	—	—	—	—	—	—	—	—	—	—	—
p=0.80	—	—	—	—	—	815	359	199	130	—	593	262	146	94	—	—	—	—	—	—	—	—	—	—	—
p=0.95	1800	444	195	110	75	75	1316	324	143	84	49	49	—	—	—	—	—	—	—	—	—	—	—	—	—
$\phi = 0.10$																									
p=0.05	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
p=0.20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
p=0.35	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
p=0.50	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
p=0.65	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
p=0.80	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
p=0.95	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—